the following parameters: $\epsilon_{r1} = \epsilon_{r2} = 12.8$, $b_1 = 0.3\ \mu$m, $b_2 = 25\ \mu$m, and $\sigma = 6 \times 10^3$ ℧/m. Such parameters could correspond to a GaAs sample at room temperature with $N_D = 1.2 \times 10^{17}$ cm^{-3}. $\bar{\beta}$ increased from 9.24 (Fig. 2) to 20.97 with a 3.7-percent increase in α to 1.96×10^4 nepers/m. Although the $\bar{\beta}$ increase due to a change in parameters is impressive for the parallel-plate waveguide at 100 GHz, the open microstrip numerical data seen here suggest that increases of 7.4:1 and 61:1 for respectively the EH_6 and EH_1 branches are required to place $\bar{\beta}$ above 8.

ACKNOWLEDGMENT

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Adjustment of In-Phase Mode in Circulators Using Turnstile Junctions

J. HELSZAJN, MEMBER, IEEE, AND J. SHARP

Abstract —**The adjustment of the counter-rotating modes of waveguide circulators using weakly magnetized turnstile junctions is fairly well understood, but some uncertainty about the definition of the in-phase mode still remains. The purpose of this paper is to remedy this situation by experimentally evaluating the in-phase eigenvalue s_0 for different filling factors and radial wavenumbers of the in-phase resonator. This is done by using the unitary condition to derive four possible relationships between the in-phase and counter-rotating eigenvalues s_0 and s_1, and the scattering variable S_{11}, and using one or another of them to form s_0. The situation for which s_0 is in anti-phase to s_1 corresponds to the first classic circulation condition of this class of device and is also derived.**

I. INTRODUCTION

The construction of the 3-port junction circulator requires the adjustment of two counter-rotating and one in-phase field patterns [1]–[3]. In the case of waveguide circulators using turnstile junctions (Fig. 1), the former ones employ quarter-wave-long resonators short-circuited at one end and open-circuited or loaded by an image wall at the other end, and the latter one usually utilizes a nearly frequency independent quasi-planar resonator which is also determined by the location of the image wall [4], [5]. The boundary conditions of the counter-rotating modes are satisfied by establishing a magnetic wall at the terminals of the junction, and that of the in-phase mode is met by placing an electric wall at the same terminals. Although the adjustment of this class of device is fairly well understood [6]–[20], there remains some uncertainty about the position of the image wall required to reconcile both boundary conditions [6], [7], [9], [10]. This difficulty is in part due to the fact that there is still no analytical description of the in-phase circuit that caters for the fringing capacitance of the resonator. The effect of a non-optimum in-phase circuit on the nature of the complex gyrator circuit of the device has been examined in [21].

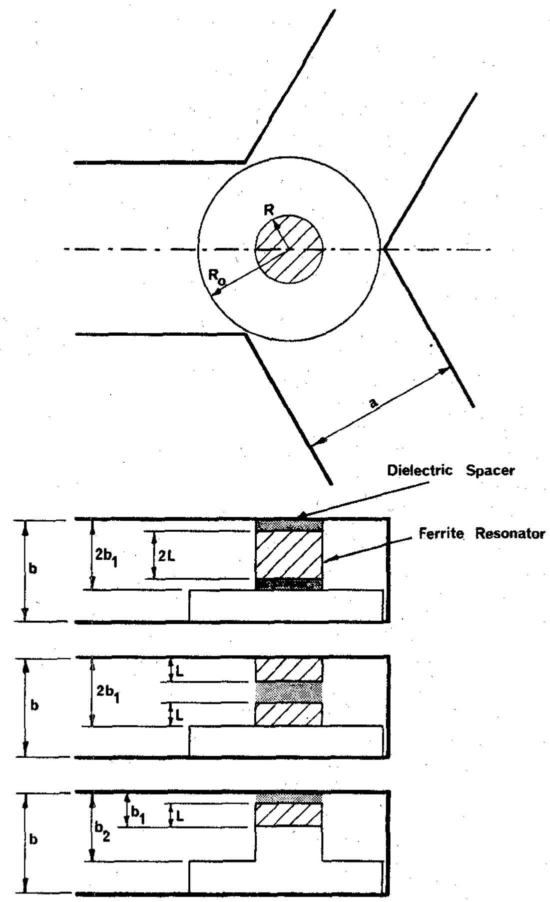

Fig. 1. Schematic diagrams of waveguide circulators using turnstile junctions.

The purpose of this paper is to remedy this situation by experimentally investigating the boundary condition of the in-phase circuit by using the unitary condition to derive four possible 1-port relationships between the reflection coefficient S_{11} and the phase angles of the in-phase and degenerate eigenvalues θ_0 and θ_1. One important conclusion of this work is that the filling factors of the single and coupled disk resonators' geometries are all but identical. The experimental relationship between the filling factor and radial wavenumber for both structures may therefore be represented by a single polynomial approximation. The data obtained here is in keeping with the semi-empirical upper

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and lower bounds previously mentioned in the literature [6], [7], [9], [10]. It is further noted that the measurement outlined in this paper not only provides a means of evaluating the in-phase eigennetwork of any weakly magnetized circulator, but it also provides a straightforward means of completely characterizing such devices.

II. Evaluation of In-Phase Eigenvalue

The phase angle of the in-phase eigennetwork of a demagnetized junction may be directly measured using the eigenvalue approach by applying equal in-phase signals at the three ports of the junction [5] or by making use of the relationship between the scattering variable S_{11} and the in-phase and degenerate counter-rotating eigenvalues s_0 and s_1 [22]

$$S_{11} = \frac{s_0 + 2s_1}{3}. \quad (1)$$

In a lossless device for which the in-phase eigennetwork may be synthesized by an open-circuited network and the degenerate counter-rotating ones by short-circuited ones

$$s_0 = 1 \cdot \exp(-j2\theta_0) \quad (2)$$

$$s_1 = 1 \cdot \exp[-j2(\theta_1 + \pi/2)] \quad (3)$$

and

$$S_{11} = |S_{11}| \cdot \exp(-j2\phi_{11}) \quad (4)$$

where

$$|S_{11}| = \frac{\text{VSWR}-1}{\text{VSWR}+1}. \quad (5)$$

The phase angle θ_0 may be evaluated using (1) by forming one of four possible relationships between the independent variables $|S_{11}|$, ϕ_{11}, and θ_1

$$2\theta_0 = \cos^{-1}(3|S_{11}|\cos 2\phi_{11} + 2\cos 2\theta_1) \quad (6)$$

$$2\theta_0 = 2\theta_1 + \pi + \cos^{-1}\left(\frac{9|S_{11}|^2 - 5}{4}\right) \quad (7)$$

$$2\theta_0 = 2\phi_{11} + \cos^{-1}\left(\frac{3|S_{11}|^2 - 1}{2|S_{11}|}\right). \quad (8)$$

The first identity is constructed by taking a linear combination of s_0 and s_0^*, and the other two are obtained by forming the products of $S_{11}S_{11}^*$ and $s_1 s_1^*$. The first relationship for θ_0 requires a knowledge of all three independent variables, whereas the second two need only a measurement of $|S_{11}|$ and a knowledge of θ_1 or ϕ_{11}.

A relationship between θ_0, θ_1, and ϕ_{11} may also be formed by eliminating $|S_{11}|$ by equating (6) and (7). The result at $2\theta_1 = \pi$ involves θ_0 and ϕ_{11} only

$$\cos^2 2\theta_0 + (4 - 4\cos^2 2\phi_{11})\cos 2\theta_0 + (4 - 5\cos^2 2\phi_{11}) = 0. \quad (9)$$

S_{11} may be evaluated by terminating the two output ports by matched loads, and s_1 may be determined by decoupling port 3 from port 1 by placing a vatiable short-circuit as port 2 [22]. The schematic diagrams of these two arrangements are illustrated in Figs. 2 and 3. The angles of the scattering variables may be located at the reference plane of the junction ($d_{s/c}$) by replacing the resonator by a metal plug. The result for ϕ_{11} is

$$2\phi_{11} = \frac{4\pi}{\lambda_g}(d_{s/c} - d_{\min,\phi_{11}}). \quad (10)$$

A similar relationship applies to $2\theta_1 + \pi$.

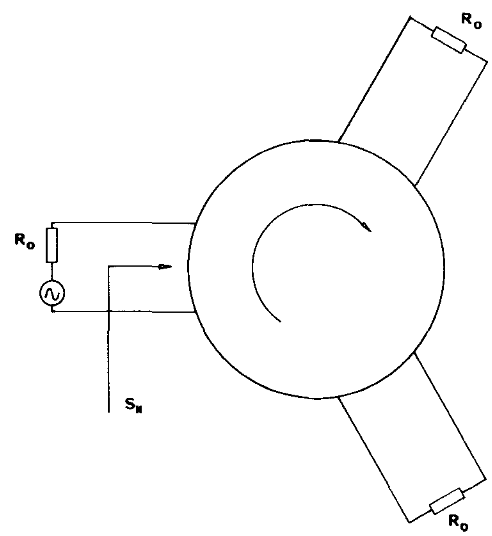

Fig. 2. Experimental arrangement for the measurement of the scattering variable S_{11}.

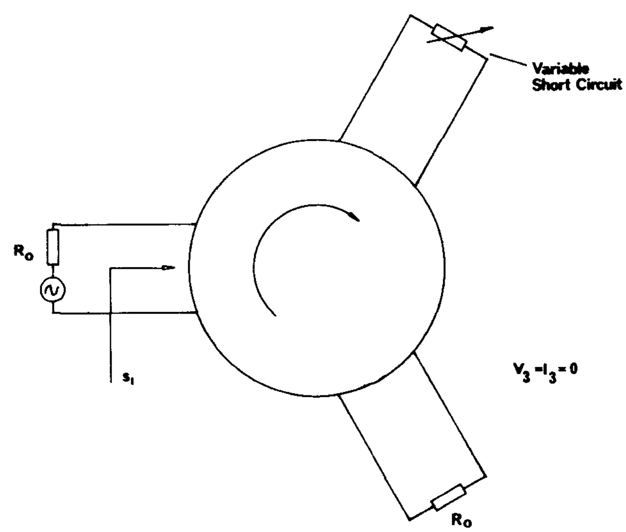

Fig. 3. Experimental arrangement for the measurement of the degenerate counter-rotating eigenvalues s_1.

Although the solution to (7) relies on two different setups, it has nevertheless been employed in preference to that in (8) because the measurement of θ_1 is unlike that of ϕ_{11}, independent of the quality of the transitions or terminations in Fig. 2.

III. In-Phase Eigennetwork

The in-phase eigennetwork is a quasi-planar resonator with virtual electric side walls and effective quasi-static dielectric constant (ϵ_{eff}) and permeability ($\mu_{d,\text{eff}}$), which may be described by [6], [7],

$$k_{\text{eff}} R_{\text{eff}} = 2.405 \quad (11)$$

$$\epsilon_{\text{eff}} \approx \frac{\epsilon_f \epsilon_d}{[(1-k)\epsilon_f + k\epsilon_d]} \quad (12)$$

$$\mu_{d,\text{eff}} \approx 1 - k(1 - \mu_d). \quad (13)$$

The filling factor k is defined by

$$k = \frac{L}{b_1} \quad (14)$$

and the demagnetized permeability μ_d is given in the usual way

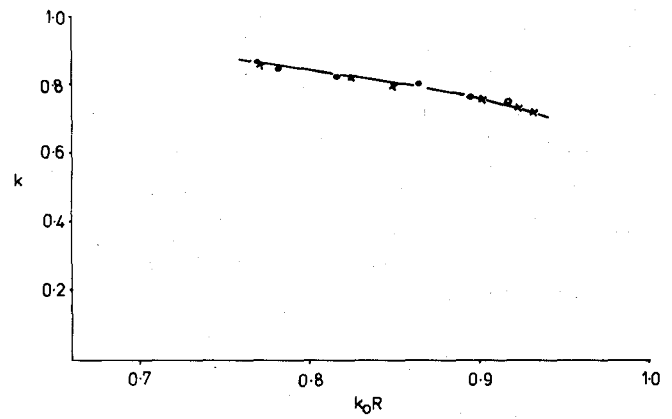

Fig. 4. Relationship between radial wavenumber $k_0 R$ and filling factor k for turnstile junctions using single and coupled disk resonators with $\epsilon_d = 1$.

by

$$\mu_d = \tfrac{1}{3} + \tfrac{2}{3}(1-p^2)^{1/2} \tag{15}$$

where

$$p = \frac{\gamma M_0}{\mu_0 \omega}. \tag{16}$$

The wavenumber k_{eff} has the usual meaning, R_{eff} is the effective radius of the resonator, ϵ_d is the relative dielectric constant of the region between the open face of the resonator and the image or waveguide wall, and ϵ_f is the relative dielectric constant of the ferrite or garnet material. γ is the gyromagnetic ratio (2.21×10^5 (rad/s)/(A/m)), M_0 is the saturation magnetization of the garnet material (T), μ_0 is the free-space permeability ($4\pi \times 10^{-7}$ H/m), and ω is the radian frequency (rad/s).

For the magnetized junction, μ_d should, strictly speaking, be replaced by the effective permeability μ_{eff} at, say, saturation

$$\mu_{\mathrm{eff}} = (1-p^2). \tag{17}$$

The nature of this eigennetwork may be understood by applying the appropriate in-phase eigenvector to the network.

Fig. 4 gives the experimental relationships between the filling factor k and the radial wavenumber $k_0 R$ for turnstile junctions using single and coupled disk resonators with $\epsilon_d = 1$. In keeping with the situation encountered with the frequencies of the counter-rotating eigennetworks, both junctions exhibit the same in-phase eigennetwork also. One suitable polynomial approximation for the filling factor of the in-phase circuit in terms of the radial wavenumber applicable to both situations is

$$k \approx 0.2196 + 2.204(k_0 R) - 1.785(k_0 R)^2, \qquad \epsilon_f = 15, \qquad 0.75 < k_0 R < 0.95 \tag{18}$$

which may be compared with the empirical values used in the past [6], [7], [9], [10].

Making use of the experimental data obtained here indicates that

$$k_{\mathrm{eff}} R_{\mathrm{eff}} \approx 0.6748 + 2.9827(k_0 R) - 2.093(k_0 R)^2, \qquad \epsilon_f = 15, \qquad 0.75 < k_0 R < 0.95. \tag{19}$$

The discrepancy between the two values of the radial variables in (11) and (19) may be understood by recognizing that the former value is based on a resonator model with ideal electric (magnetic) walls, whereas in practice the boundary conditions consist of imperfect electric (magnetic) walls. This EM problem is outside the remit of this work.

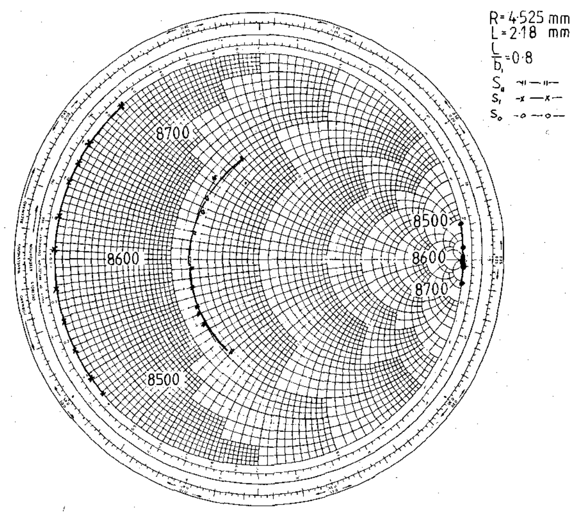

Fig. 5. Smith Chart of s_0, s_1, and S_{11} variables.

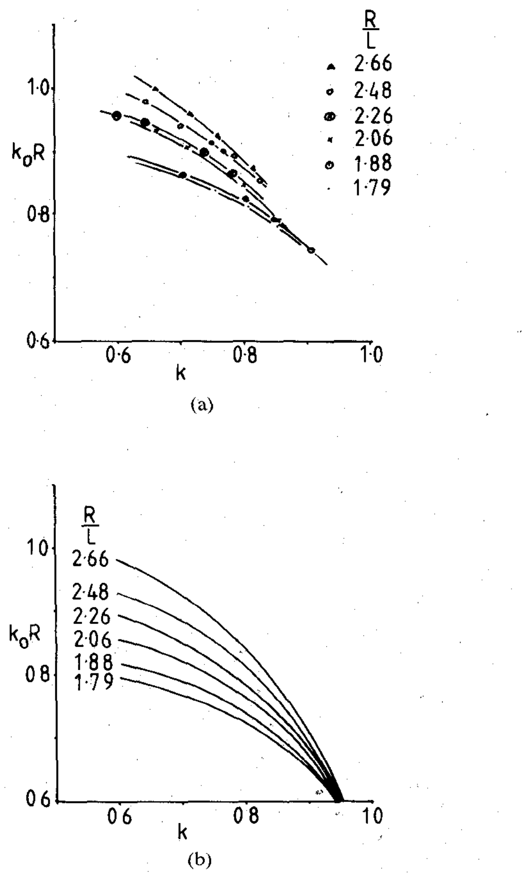

Fig. 6. (a) Experimental and (b) theoretical mode chart for counter-rotating modes of turnstile junctions using single and coupled disk resonators.

IV. Adjustment of First Circulation Condition in Circulators Using Turnstile Junctions Employing Single and Coupled Disk Resonators

Since the determination of the phase angle θ_0 also requires a knowledge of θ_1, the evaluation of the former quantity affords the experimental means for the latter to be obtained as well, and furthermore, the necessary and sufficient conditions of the first circulation condition to be formed. Fig. 5 indicates the frequency responses of s_0, s_1, and S_{11} for a turnstile junction using a single resonator with $k_0 R = 0.816$ and $k = 0.80$. The condition $s_1 = 1$ provides a simple statement of the resonant frequency of the

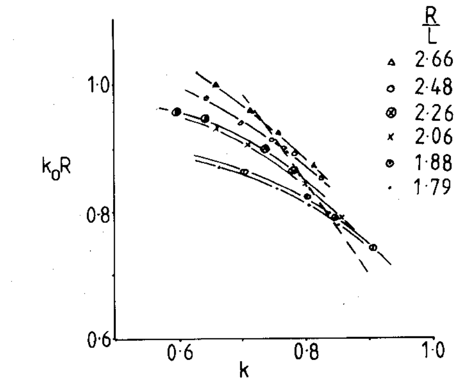

Fig. 7. First circulation solution for turnstile junctions using single and coupled disk resonator.

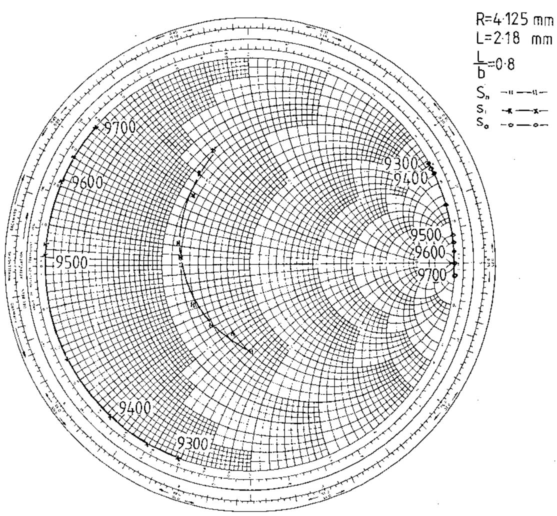

Fig. 8. Effect of mistuning in-phase eigennetwork.

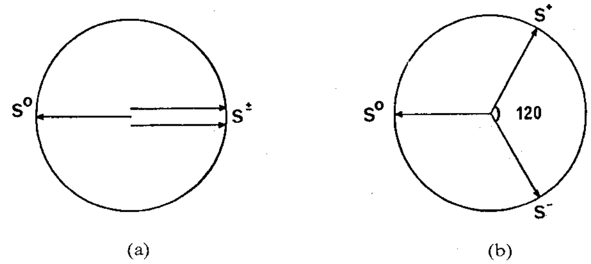

Fig. 9. (a) Eigenvalue diagram of 3-port junction with $s_0 = 1$ and $s_1 = -1$. (b) Eigenvalue diagram of ideal 3-port junction circulator.

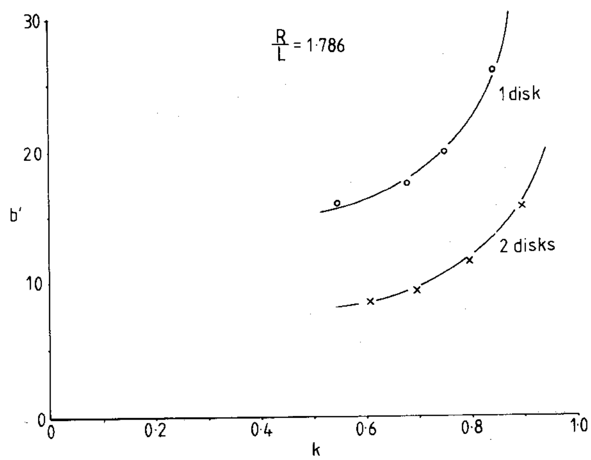

Fig. 10. Frequency response of radial line coupled turnstile junction using single resonator.

degenerate counter-rotating eigennetworks. Fig. 6 summarizes this result. The necessary and sufficient conditions for the adjustment of this class of circulator is now met by superimposing the data in Figs. 4 and 6 in the manner illustrated in Fig. 7. The intersection of these two relationships gives the required one between θ_1 and θ_0. This result, strictly speaking, applies to a garnet resonator with a magnetization of 0.1600 T and a relative dielectric constant of 15.0. If k_0R is taken as the independent variable, then k and R/L are the dependent ones. The effect of mistuning the in-phase eigennetwork is separately illustrated in Fig. 8. Obviously, the reference plane of S_{11} is in this situation incompatible with the synthesis of high-quality quarter-wave coupled junction circulators [21].

The situation in Fig. 5 is sometimes displayed on a unit circle in the manner indicated in Fig. 9(a). Since for this class of junction it is now merely necessary to split the degeneracy between the counter-rotating eigenvalues in the manner indicated in Fig. 9(b) in order to realize an ideal circulator, this condition is sometimes referred to as the first circulation condition. However, this is not a general result but merely a special case of the more general boundary condition obtained by setting the imaginary part of the complex gyrator immittance to zero. It is obviously an adequate boundary condition for the problem at hand. It is also noted from the data in Fig. 5 that the frequency variation of the in-phase eigennetwork may indeed be neglected compared to those of the degenerate counter-rotating ones as is often assumed.

The first circulation adjustment described here has been incorporated in the experimental construction of one circulator in WR90 waveguide using a radial coupled turnstile junction employing a single disk resonator. Fig. 10 depicts the frequency response of the device. The details of the second circulation condition (radial transformer, magnetic variables, susceptance slope parameters, etc.) are in keeping with the material in [20]. Those of the first circulation condition are in very close agreement with the data developed here. No tuning whatsoever was utilized in obtaining this result.

V. Susceptance and Reactance Slope Parameters of In-Phase and Counter-Rotating Eigennetworks

The reactance slope parameter of the in-phase eigennetwork (and for completeness that of the degenerate counter-rotating eigennetworks) is of considerable interest in determining the second circulation condition of the device. These two quantities may be evaluated from data such as that in Fig. 5, by using the following standard relationships:

$$x' = \frac{\omega}{2Z_0} \frac{\delta x}{\delta \omega}\bigg|_{\omega_0} \tag{20}$$

$$b' = \frac{\omega}{2G_0} \frac{\delta B}{\delta \omega}\bigg|_{\omega_0}. \tag{21}$$

This result indicates that the frequency variation of s_0 may be neglected compared to that of s_1 and that it may, indeed, be approximated by a frequency independent short-circuit boundary condition at the terminals at the junction as is sometimes assumed [19], [20].

Fig. 11 indicates the experimental relationship between the normalized susceptance slope parameter b_1' and the filling factor k for single and coupled disk resonators. It is seen from this data that the susceptance slope parameter of turnstile junctions employing coupled disk resonators is essentially half that of junc-

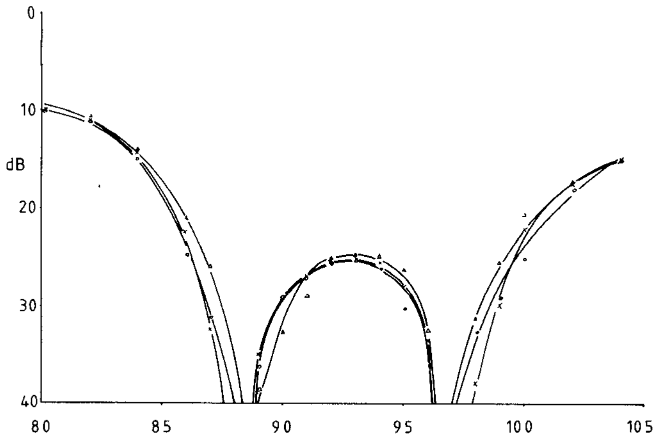

Fig. 11. Susceptance slope parameter of degenerate counter-rotating eigennetworks for single and coupled disk resonators.

tions using a single resonator in keeping with some experimental work elsewhere [10], [11].

The experimental data obtained here on the degenerate counter-rotating eigennetworks are deemed, if anything, more accurate than similar previously reported material on them in that they do not rely on the assumption that the in-phase eigennetwork has been idealized by a frequency independent short-circuit boundary condition.

VI. Conclusions

An important quantity in the physical design of waveguide circulators using turnstile junctions is the adjustment of the in-phase mode. This paper describes four simple measurements which permits this adjustment to be determined. The relationship between the radial wavenumber, the filling factor, and the dielectric constant of the region between the open face of the ferrite region and the image wall is given in polynomial form for both single and coupled disk resonators and is in keeping with the qualitative appreciation of this eigennetwork. One important conclusion of this work is that the in-phase eigennetworks of turnstile junctions using either resonators are identical. The experimental procedure outlined here also permits the accurate construction of the mode chart of the degenerate counter-rotating modes and a complete description of the first circulation relationship of this class of device.

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Letters

Comments on "Optical Injection Locking of BARITT Oscillators"

A. J. SEEDS, MEMBER, IEEE, AND J. R. FORREST, MEMBER, IEEE

We were interested to read the above short paper,[1] but would like to point out that experimental results on the optical injection locking of IMPATT oscillators have indeed been obtained. Over the last few years, reports of such experiments have appeared in the literature from groups in both Europe [1], [2] and the U.S.A. [3].

While optically injection-locked BARITT oscillators may find application in microwave receivers, their use as transmitter elements in phased-array radar systems, as proposed by the authors of the above paper,[1] would seem to be limited by their rather restricted output power capabilities [4]. It should also be noted that optically generated carriers in IMPATT diodes benefit from the avalanche multiplication inherent in the operation of the device, leading to a considerable improvement in locking range for a given optical power [2]. Such a gain mechanism is not available in the BARITT diode.

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